

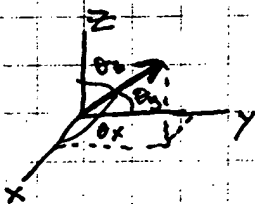
In this latter picture a para corresponds to a ray ending up exactly on lattice position removed from its origin position.

- ① pick a direction vector
- ② calculate reflection off 3 sets of orthogonal plane
- ③ calculate travel distance between successive hits on a single set of planes
- ④ assign a loss/cm number

this looks amenable to an analytic treatment.

5-Sep-97

use direction cosines to parameterize ray direction

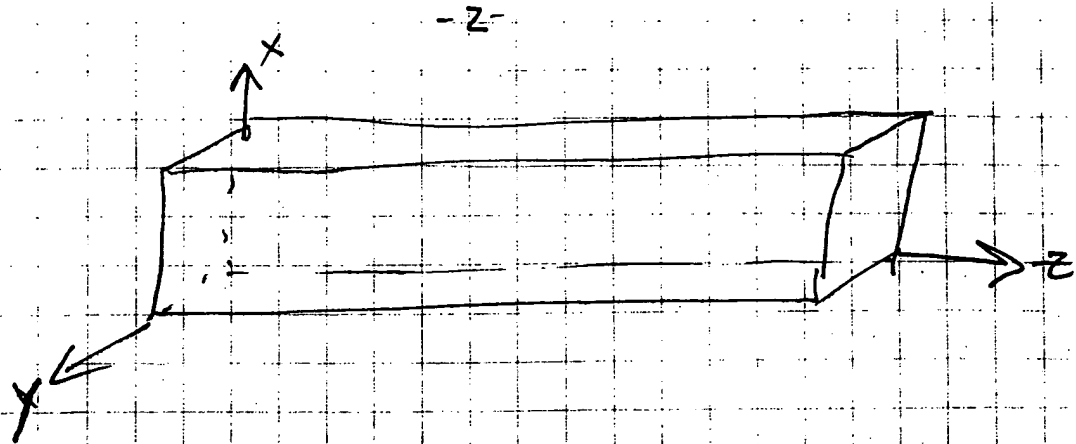


$$(\cos \theta_x, \cos \theta_y, \cos \theta_z) = \frac{(RND_1, RND_2, RND_3)}{\sqrt{RND_1^2 + RND_2^2 + RND_3^2}}$$

Let  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$  denote slab dimensions or plane spacing.

6-Sep-97

From this point of view it doesn't matter what position a ray is launched from, only its direction, because launch position has no impact on spacing between plane strikes.



Treat cants on surface perturbatively.

2 questions:

① How big do cants have to be to eliminate all parasites?

② For a rectangular slab, how close to slab index does cladding index have to be to eliminate all parasites.

This question will be easiest to answer for a zero loss parasite

set  $z$  face incident angle equal to  $\theta_{crit} = \sin^{-1}(\frac{1}{n_s})$  and the  $x$  face hit and  $y$  face hit also =  $\theta_{crit} = \sin^{-1}(\frac{n_c}{n_s})$

make this argument more rigorous!

Now work with direction cosines

$$\theta_{c1} = \sin^{-1}(\frac{1}{n_s})$$

$$\theta_{c2} = \sin^{-1}(\frac{n_c}{n_s})$$

$$\cos \theta_{c1} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

$$\cos \theta_{c2} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos^2 \theta_{c1} + 2 \cos^2 \theta_{c2} = 1$$

$$\frac{n_s^2 - 1}{n_s^2} + \frac{2(n_s^2 - n_c^2)}{n_s^2} = 1$$

$$n_s^2 - 1 + 2n_s^2 - 2n_c^2 = n_s^2$$

$$2(n_s^2 - n_c^2) = 1$$

$$n_s^2 - n_c^2 = \frac{1}{2}$$

when can this no longer be solved

$$n_c = \sqrt{n_s^2 - \frac{1}{2}}$$

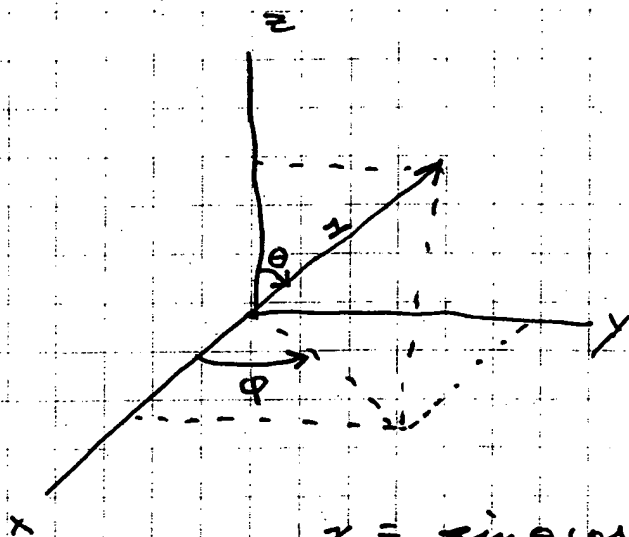
$$n_c = \sqrt{1.82^2 - \frac{1}{2}} = \underline{\underline{1.677}}$$

for  $n_c > 1.677$  no zero loss  
parasitics exist!

↳ this agrees with  
code: Slab ASE 01.XCL  
prediction.

Question 2 will be easiest to answer numerically by  
finding the angular width over which  
a parasitic exists for given gain and  
cladding indices.

7-sep-97



$$\begin{aligned}
 x &= \sin \theta \cos \phi = \cos \theta_x \\
 y &= \sin \theta \sin \phi = \cos \theta_y \\
 z &= \cos \theta = \cos \theta_z
 \end{aligned}$$

$$\begin{aligned}
 \theta_x &< \theta_{x \text{ crit}} \\
 \theta_y &< \theta_{y \text{ crit}} \\
 \theta_z &< \theta_{z \text{ crit}}
 \end{aligned}$$

to avoid 0-loss  
parasitics

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\begin{aligned}
 \cos \theta_x &> \cos \theta_{x \text{ crit}} \\
 \cos \theta_y &> \cos \theta_{y \text{ crit}} \\
 \cos \theta_z &> \cos \theta_{z \text{ crit}}
 \end{aligned}$$

to avoid  
0-loss  
parasitics

$$\begin{aligned}
 \sin \theta_{x \text{ crit}} &= \frac{n_c}{n_s} \\
 \sin \theta_{y \text{ crit}} &= \frac{n_c}{n_s} \\
 \sin \theta_{z \text{ crit}} &= \frac{1}{n_s}
 \end{aligned}$$

$$\begin{aligned}
 \theta_{x \text{ crit}} &= \arcsin \left( \frac{n_c}{n_s} \right) \\
 \theta_{y \text{ crit}} &= \arcsin \left( \frac{n_c}{n_s} \right) \\
 \theta_{z \text{ crit}} &= \arcsin \left( \frac{1}{n_s} \right)
 \end{aligned}$$

$$\cos \theta_x > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_y > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_z > \frac{\sqrt{n_s^2 - 1}}{n_s}$$

to avoid  
0-loss  
parasiti

$1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$  and this must be greater than

$$1 > \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

to avoid  
0-loss  
parasitic

$$1 > \frac{3n_s^2 - 2n_c^2 - 1}{n_s^2}$$

$$n_s^2 > 3n_s^2 - 2n_c^2 - 1$$

$$1 > 2(n_s^2 - n_c^2)$$

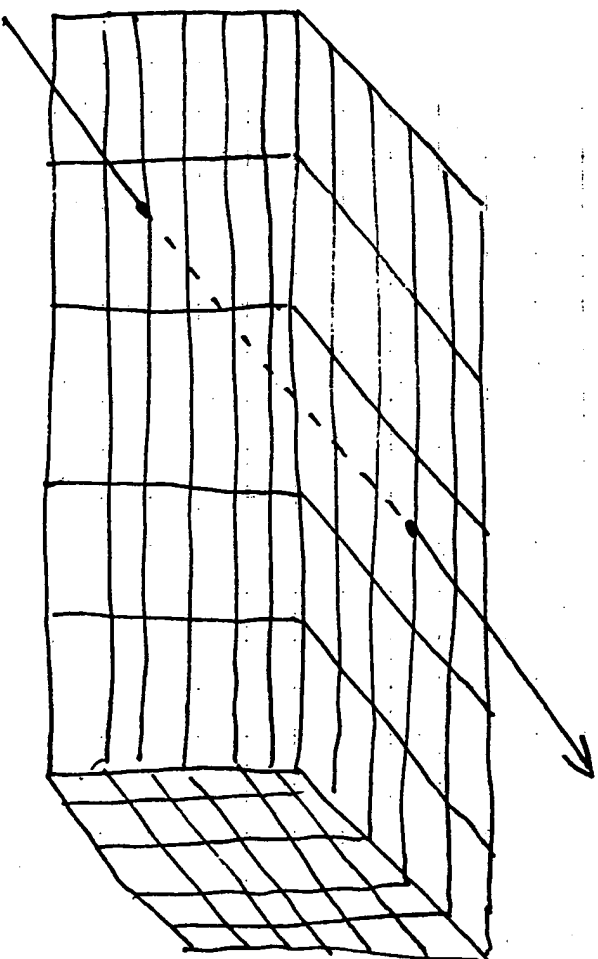
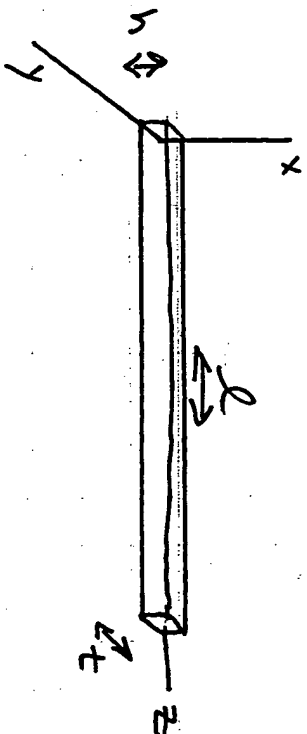
$$\frac{1}{2} > n_s^2 - n_c^2$$

$$n_c^2 > n_s^2 - \frac{1}{2}$$

$$n_c > \sqrt{n_s^2 - \frac{1}{2}} = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

$n_s$  is stat index  
 $n_c$  is crit index

Paraxial propagation directions can be represented using a method of images construction



fill space  
using slab-  
and  
slab-images

- define arbitrary ray direction using direction cosines  $(\cos \theta_x, \cos \theta_y, \cos \theta_z)$
- gain of ray in given in repeat/can be by

$$\delta = \frac{\ln(\text{Ref}_x)}{(h/\cos \theta_x)} \frac{\ln(\text{Ref}_y)}{(t/\cos \theta_y)} \frac{\ln(\text{Ref}_z)}{(L/\cos \theta_z)} \alpha$$

where:  $\text{Ref}_i$  is the reflection coefficient for i-oriented planes  
 $\alpha$  is slab specific gain loading

zero-loss paraitics correspond to those ray directions that are confined by TIR at all three sets of planes

$$\left\{ \begin{array}{l} \text{TIR} \\ \text{condition} \end{array} \right. \quad \left\{ \begin{array}{l} \cos \theta_x < \cos \theta_{x\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_y < \cos \theta_{y\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_z < \cos \theta_{z\text{-crit}} = \frac{\sqrt{n_s^2 - 1}}{n_s} \end{array} \right.$$

where:  
 $n_s$  = slab index  
 $n_c$  = coating index

• Since  $1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$ , zero loss paraitics exist when

$$1 < \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

or

$$n_c < \sqrt{n_s^2 - 1/2}$$

zero-loss paraitics can be completely suppressed by choosing a cladding with refractive index large enough

$$n_c > \sqrt{n_s^2 - 1}$$